UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/02 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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UNIVERSITY of CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Equate radial forces when string slackens:	$mv^2/a = mg\cos 60°$	M1		
	Rearrange to obtain v:	$v = \sqrt{(\frac{1}{2}ga)}$ **A.G.**	A1		
	Use conservation of energy:	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga(1+\cos 60°)$	M1 A1		
	Substitute for v and rearrange to obtain u:	$u^2 = \frac{1}{2}ga + 3ga,\ u = \sqrt{(7ga/2)}$			
		or $1.87\sqrt{(ga)}$ *or* $5.92\sqrt{a}$	A1	(5)	**[5]**
2	Find MI of wheel:	$I = \frac{1}{2}\times 6\times 0.25^2$ [$= 3/16 = 0.1875$]	B1		
	Find angular deceln. from ω, t (ignoring sign):	$\lvert d^2\theta/dt^2\rvert = \omega / t = 2/5$ *or* 0.4	M1 A1		
	Use moment eqn to find magnitude of braking force:	$F = I\lvert d^2\theta/dt^2\rvert / r = 3/10$ *or* 0.3 [N]	M1 A1	(5)	
	Find angle turned by wheel:	$\theta = \omega^2/2\lvert d^2\theta/dt^2\rvert$ [$=2^2/(2\times 0.4)$]			
		or $\frac{1}{2}\omega t$ [$= \frac{1}{2}\times 2\times 5$]			
		or $\omega t + \frac{1}{2}\, d^2\theta/dt^2\, t^2$ [$= 10 - 5$]			
		or $\frac{1}{2} I\,(d\theta/dt)^2 / rF$			
		[$= \frac{1}{2}(3/16)\,2^2 / (0.25\times 0.3)$]	M1		
		$= 5$ [rad]	A1	(2)	**[7]**
3	Use conservation of momentum:	$mv_A + 3mv_B = mu$	M1		
	Use Newton's law of restitution:	$v_A - v_B = -eu$	M1		
	Solve for v_A:	$v_A = \frac{1}{4}(1-3e)\,u$	M1 A1		
	Use $e > \frac{1}{3}$ to find direction of A:	$1 < 3e$ so $v_A < 0$ **A.G.**	B1	(5)	
	Find speed of B before striking barrier:	$v_B = \frac{1}{4}(1+e)\,u$	A1		
	Find rel. speed or ratio of speeds after collision:	$e\,v_B + v_A = \frac{1}{4}(1-e)^2 u$			
	or	$-v_A/e\,v_B = 1 - (1-e)^2/e(1+e)$	M1		
	Derive condition for subsequent collision:	$e\,v_B > -v_A$ unless $e = 1$ **A.G.**	A1	(3)	**[8]**

Question Number	Mark Scheme Details			Part Mark	Total
4	Find MI of rod and ring about A:	$I = ⅓\, 2ma^2 + 2ma^2 + ma^2$	M1		
	Use conservation of energy:	$½\, I\, (d\theta/dt)^2 = 3mga \sin\theta$	M1		
	Substitute $I = (11/3)\, ma^2$:	$a\,(d\theta/dt)^2 = (18/11)\,g \sin\theta$ **A.G.**	A1	(3)	
	Resolve forces on ring along rod (ignore sign of F):	$F = mg \sin\theta + ma\,(d\theta/dt)^2$	M1		
	Substitute for $d\theta/dt$:	$F = (29/11)\, mg \sin\theta$ **A.G.**	A1		
	Resolve forces on ring normal to rod (ignore signs):	$R = mg \cos\theta - ma\, d^2\theta/dt^2$	M1		
	Find $d^2\theta/dt^2$ by differentiating eqn above:	$a\, d^2\theta/dt^2 = (9/11)\,g \cos\theta$	M1 A1		
	Combine to give magnitude of normal force:	$R = (2/11)\, mg \cos\theta$ **A.G.**	A1	(6)	
	Relate μ to θ:	$\mu = F/R = (29/2) \tan\theta$	M1		
	Find θ:	$\theta = \tan^{-1}(2\mu/29)$	A1	(2)	**[11]**
5	Equate moments of AB and BC about A:	$Wa \sin\alpha = Wa\,(\cos\alpha - 2\sin\alpha)$	M1 A1		
	Evaluate α:	$\tan\alpha = ⅓,\ \alpha = 18{\cdot}4°$ **A.G.**	A1	(3)	
	Take moments about B for rod BC:	$T\, 2a \cos 45° = Wa \cos\alpha$	M1 A1		
	Find tension T in form kW:	$T = (\cos 18{\cdot}4° / \sqrt{2})\, W = 0{\cdot}671\, W$	A1	(3)	
	EITHER: Resolve vertically for rod BC:	$R_V = W - T \sin(45° + \alpha)$			
	(ignore signs of all components R_V etc)	$= W - T \sin 63{\cdot}4°$ [$= 0{\cdot}400\, W$]	M1 A1		
	Resolve horizontally for rod BC:	$R_H = T \cos(45° + \alpha)$ [$= 0{\cdot}300\, W$]	M1 A1		
	OR: Resolve along AB for rod BC:	$R_{AB} = W \cos\alpha - T \sin 45°$			
	(or take moments about C)	[$= 3W/2\sqrt{10} = 0{\cdot}474\, W$]	(M1 A1)		
	Resolve along BC for rod BC:	$R_{BC} = W \sin\alpha - T \cos 45°$			
		[$= -W/2\sqrt{10} = -0{\cdot}158\, W$]	(M1 A1)		
	Combine components, e.g $\sqrt{(R_V^2 + R_H^2)}$:	$R = 0{\cdot}5\, W$	M1 A1	(6)	**[12]**

Question Number	Mark Scheme Details			Part Mark	Total
6	Find sample mean (2 dp required):	$\bar{x}$ = 60·18 / 6 = 10·03	B1		
	Estimate population variance to 3 sf (allow biased here: 0·00107 *or* 0·0327^2):				
		s^2 = (603·6118 – 60·18^2/6) / 5 = 0·00128 *or* 0·0358^2	B1		
	Use consistent formula for C.I. with any t value:	$\bar{x} \pm t\sqrt{(s^2/6)}$	M1√		
	Use of correct tabular value:	$t_{5,\,0.995}$ = 4·03[2]	*A1		
	C.I. correct to 2 dp in cm (dep *A1):	10·03 ± 0·06 *or* [9·97, 10·09]	A1	(5)	
	Find CI for mean circumference to 1 dp in cm:	10·03π ± 0·06π			
		= 31·5 ± 0·2 *or* [31·3, 31·7]	B1	(1)	**[6]**
7	Find P($X \geq$ ¾):	1 – F(¾) = 1 – ½[(¾)^3 + 1]	M1		
		= 37/128 *or* 0·289	*A1		
	State deduction about upper quartile Q_3 (dep *A1):	Q_3 > ¾	B1	(3)	
	Express cum. dist. fn. G of Y in terms of X:	$G(y) = P(Y \leqslant y) = P(-\sqrt{y} \leqslant X \leqslant \sqrt{y})$	M1 A1		
	Relate to F:	$= F(\sqrt{y}) - F(-\sqrt{y})$	M1		
	Simplify:	$= ½[(\sqrt{y})^3 + 1] - ½[(-\sqrt{y})^3 + 1]$			
		$= y^{3/2}$	A1		
	State G in full (√ on previous result):	0 ($y < 0$), $y^{3/2}$ ($0 \leqslant y \leqslant 1$), 1 ($y > 1$)	A1√	(5)	**[8]**
8	State if assumption necessary with valid reason:	No, since large samples (A.E.F.)	B1	(1)	
	State another valid assumption:	Samples must be random (A.E.F.)	B1		
	State hypotheses:	H_0: $\mu_c - \mu_f = 5$, H_1: $\mu_c - \mu_f < 5$	B1		
	Find z (A.E.F.):	z = (19·14 – 15·36 – 5) /			
		√(20·54/75 + 9·84/75)	M1 A1		
		= –1·22/√(2×15·19/75) = –1·92	*A1		
	Use correct tabular value of z:	$z_{0.95}$ = 1·64[5]	*B1		
	Consistent conclusion (A.E.F.; A1 dep *A1, *B1):	Does exceed by less than 5 kg.	M1√ A1	(8)	**[9]**

Question Number	Mark Scheme Details			Part Mark	Total
9	Show how value is found (allow M1 if 100 omitted):	$100\ {}^4C_2\ 0{\cdot}4^2 \times 0{\cdot}6^2 = 34{\cdot}56$ **A.G.**	M1 A1	(2)	
	State (at least) null hypothesis:	H_0: B(4, 0·6) fits data (A.E.F.)	B1		
	Combine adjacent cells since exp. value < 5:	*O*: 14 27 49 10			
		E: 17·92 34·56 34·56 12·96	*M1		
	Calculate value of χ^2 (to 2 dp ; A1 dep *M1):	$\chi^2 = 9{\cdot}22$	M1 A1		
	Compare with consistent tabular value (to 2 dp):	${\chi_{3,\,0.95}}^2 = 7{\cdot}815$ (cells combined)			
		${\chi_{4,\,0.95}}^2 = 9{\cdot}488$ (not combined)	B1		
	Valid method for reaching conclusion:	Reject H_0 if χ^2 > tabular value	M1		
	Correct conclusion (A.E.F., requires correct values):	9·22 > 7·81[5] so distn. does not fit	*A1		
	State valid deduction (dep *A1; allow $p \neq 0{\cdot}6$):	Prob. of faulty chips ≠ 0·6 (A.E.F.)	B1	(8)	**[10]**
10	Use geometric distribution [with $p = 3/20 = 0{\cdot}15$]		M1		
	Find prob. of missing in 5 shots:	q^5 *or* $1 - p\,(1 + q + q^2 + q^3 + q^4)$	M1		
	(q^6 *or* 0·377 earns M1 A0)	$= 0{\cdot}85^5 = 0{\cdot}444$	A1	(3)	
	Separate probabilities of hits:	$P(Y = r) = P(2 \text{ hits in } r{-}1 \text{ shots}) \times$			
		$P(\text{hit on } r^{\text{th}} \text{ shot})$	M1		
	Substitute using geometric distribution:	$= {}^{r-1}C_2 \times 0{\cdot}15^2 \times 0{\cdot}85^{r-3} \times 0{\cdot}15$	M1		
	Replace ${}^{r-1}C_2$: **A.G.**	$= \tfrac{1}{2}(r-1)(r-2)\ 0{\cdot}15^3 \times 0{\cdot}85^{r-3}$	A1	(3)	
	Simplify $P(Y = r{+}1) / P(Y = r)$:	$0{\cdot}85\, r/(r-2)$ *or* $17r/20(r-2)$	B1		
	Find required values of r:	$17r < 20(r-2)$	M1		
		$r > 40/3$ *or* 13·3 *or* 13	A1		
	Deduce most probable value of Y:	14 (√ on values of r)	A1√	(4)	**[10]**

Question Number	Mark Scheme Details			Part Mark	Total
11 **EITHER**	Find extension e_0 at equilibrium position:	$4mge_0/l = mg, \quad e_0 = \frac{1}{4}l$	B1		
	Apply Newton's law at general point:	$m\,d^2x/dt^2 = mg - 4mg(e_0 + x)/l$	M1 A1		
	Combine:	$d^2x/dt^2 = -4gx/l$ **A.G.**	A1	(4)	
	Find SHM amplitude A (or A^2) from initial speed:	$gl = \omega^2A^2, \quad A^2 = \frac{1}{4}l^2$ *or* $A = \frac{1}{2}l$	M1 A1		
	Find speed v when string's length is l:	$v^2 = \omega^2(A^2 - e_0^2)$			
		$= (4g/l)(\frac{1}{4}l^2 - l^2/16), \; v = \sqrt{(\frac{3}{4}gl)}$	M1 A1	(4)	
	Find time from $x = A$ to $x = -e_0$ by e.g.:	$t_1 = \omega^{-1}\cos^{-1}(-e_0/A)$			
		or $\frac{1}{2}T - \omega^{-1}\cos^{-1}(e_0/A)$			
		or $\omega^{-1}\sin^{-1}(-e_0/A) - \frac{1}{4}T$	M1		
	Substitute for ω, e_0, A, T:	$t_1 = \frac{1}{2}\sqrt{(l/g)}\cos^{-1}(-\frac{1}{2})$			
		or $\frac{1}{2}\pi\sqrt{(l/g)} - \frac{1}{2}\sqrt{(l/g)}\cos^{-1}(\frac{1}{2})$			
		or $\frac{1}{2}\sqrt{(l/g)}\sin^{-1}(-\frac{1}{2}) - \frac{1}{4}\pi\sqrt{(l/g)}$	A1		
	Simplify:	$t_1 = (\pi/3)\sqrt{(l/g)}$	A1		
	Find further time to rest (A.E.F.):	$t_2 = v/g = \sqrt{(\frac{3}{4}l/g)}$ *or* $\frac{1}{2}\sqrt{(3l/g)}$	M1 A1		
	Combine times:	$t_1 + t_2 = (\frac{1}{3}\pi + \sqrt{3}/2)\sqrt{(l/g)}$ **A.G.**	A1	(6)	**[14]**

Question Number	Mark Scheme Details			Part Mark	Total
11 OR	State what is minimised (A.E.F.):	Sum of squares of residuals	B1		
	Show residuals on copy of diagram:	Verticals between points and line	B1	(2)	
	State answer with valid reason (A.E.F.) e.g.:	No, since points not collinear	B1	(1)	
	(i) Calculate correlation coefficient:				
	$r = (1022{\cdot}15 - 340 \times 22{\cdot}41/8)/\sqrt{\{(15\,500 - 340^2/8)(67{\cdot}65 - 22{\cdot}41^2/8)\}}$		M1 A1		
		$= 69{\cdot}725/\sqrt{(1050 \times 4{\cdot}874)} = 0{\cdot}975$	*A1		
	State valid comment on diagram (A.E.F.):	Points lie close to a straight line	B1√		
	(B0 for 'points are nearly correlated')			(4)	
	(ii) State valid reason (A.E.F.; dep *A1):	0·975 closer to 1 than 0·965	B1		
	Find coefficient b in regression line for y:	$b = (1022{\cdot}15 - 340 \times 22{\cdot}41/8)/(15\,500 - 340^2/8)$			
		$= 69{\cdot}725/1050 = 0{\cdot}0664$	M1 A1		
	Find equation of regression line:	$\sqrt{z} = b(v - 42{\cdot}5) + 2{\cdot}80125$			
		$= 0{\cdot}0664\,v - 0{\cdot}0207$[5]			
		or $0{\cdot}066\,v - 0{\cdot}021$	M1 A1	(5)	
	(iii) All answers A.E.F.:				
	EITHER: State physical significance:	Equivalent to $\sqrt{z}$ for zero speed	B1		
	OR: State expected relationship:	Constant force so $v^2 \propto z$	(B1)		
	Valid comment, √ on 0·021:	0·021 *or* $0{\cdot}021^2 \approx 0$ as expected			
		or $\neq 0$ suggests error in data	B1	(2)	**[14]**